Searching for Subspace Trails and Truncated Differentials

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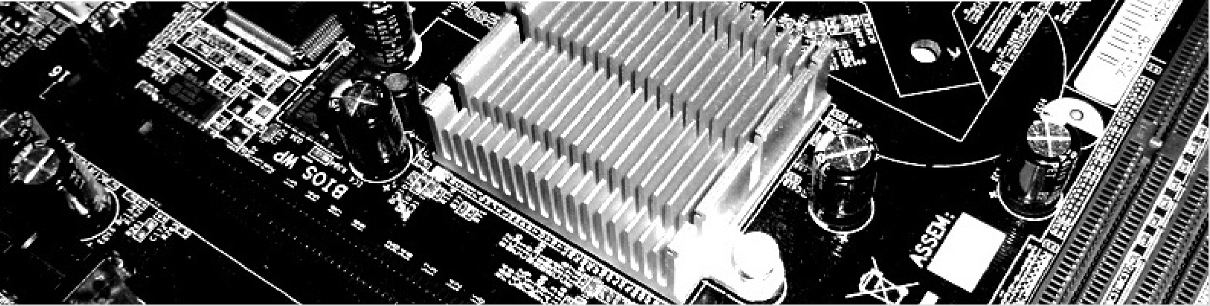
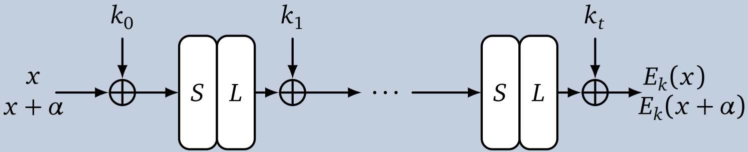


Differential Cryptanalysis

SPN Cipher



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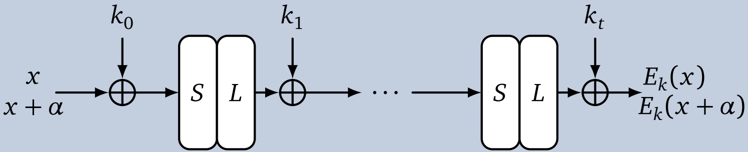
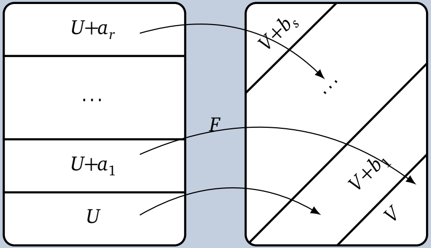


Definition [Knu94; BLN14]

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. A *truncated differential* of probability one is a pair of affine subspaces $U{+}s$ and $V{+}t$ of $\mathbb{F}_2^n$, s. t.

$$\forall u \in U : \forall x \in \mathbb{F}_2^n : \qquad F(x) + F(x+u+s) \in V{+}t$$

Main Idea



Structural Attacks

Subspace Trail Cryptanalysis

RUB

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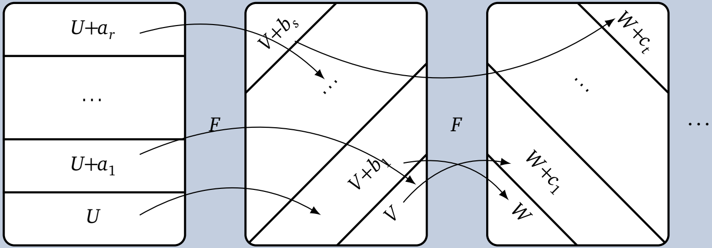

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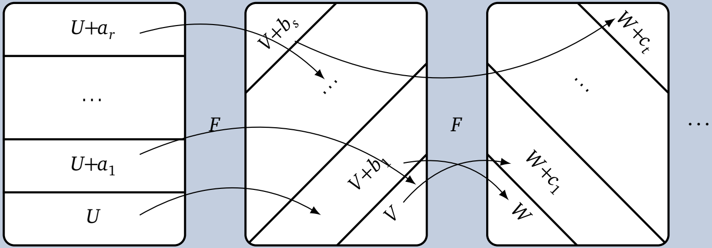

Subspace Trail Cryptanalysis [GRR16] (Last Year's FSE)

Let $U_0, \dots, U_r \subseteq \mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. We write $U_0 \xrightarrow{F} \cdots \xrightarrow{F} U_r$, iff

$$\forall a \in U_i^{\perp} : \exists b \in U_{i+1}^{\perp} : \qquad F(U_i + a) \subseteq U_{i+1} + b$$

Outline

RUB

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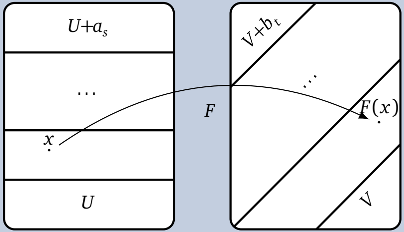
Intuition

The Image of the Derivative is in the Subspace

Lemma

Let $U \xrightarrow{F} V$ *be a subspace trail. Then for all* $u \in U$ *and all* x*:* $F(x) + F(x+u) \in V$.

Proof



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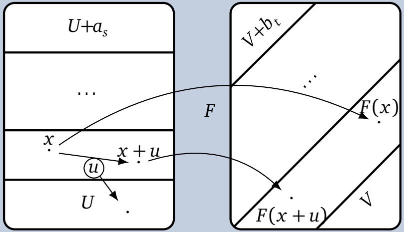

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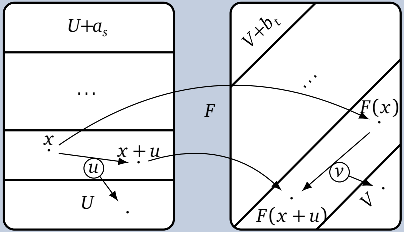

Link to Truncated Differentials

Direct consequence from above Lemma

Theorem (Subspaces Trails are Truncated Differentials with probability one)

Let $U \xrightarrow{F} V$ be a subspace trail.
Then $U+0$ and $V+0$ form a truncated differential with probabiliy one.

Subspace Trails are thus a special case of truncated differentials.

Provable Resistant against Subspace Trails

How to search efficiently for Subspace Trails?

Security against Subspace Trails?

Given the round function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ of an SPN cipher, prove the resistance against subspace trail attacks!

1

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Already for initially one-dimensional subspaces there are $2^n - 1$ possibilities.

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Can't we just activate a single S-box and check to what this leads us?

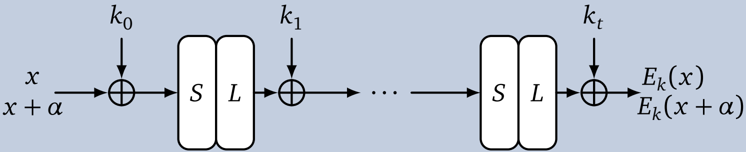
The short answer is:
No![1]

[1] The long answer is: Read our paper ☺

Approach to the Algorithm

How to reduce the number of starting points?

SPN Cipher



Easy parts

- Given a starting subspace, computing the trail is easy.
- The effect of the linear layer L to a subspace U is clear:

$$U \xrightarrow{L} L(U)$$

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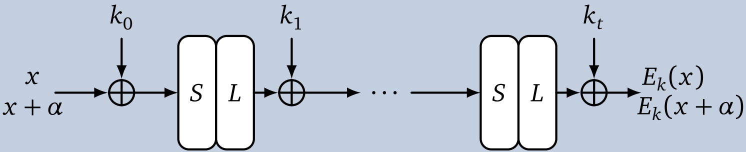

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S-box: First Observation

For an S-box S and $U \xrightarrow{S} V$, because of the above lemma, $\forall x \in \mathbb{F}_2^n$ and $\forall u \in U$:

$$S(x) + S(x+u) \in V$$

$$\Leftrightarrow \langle \alpha, S(x) + S(x+u) \rangle = 0 \quad \forall \alpha \in V^{\perp}.$$

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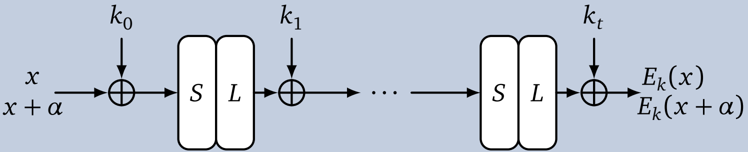

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$$\Leftrightarrow \langle \alpha, S(x) + S(x+u) \rangle = 0 \quad \forall \alpha \in V^\perp.$$

By definition, $V^\perp$ is the set of zero-linear structures of S.

Possibility I

The short one

RUB

Theorem

Let $F : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ be an S-box layer that applies k S-boxes with no non-trivial linear structures in parallel.

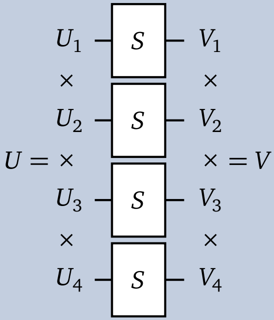
Then every essential subspace trail $U \xrightarrow{F} V$ is of the form

$$U = V = U_1 \times \cdots \times U_k,$$

where $U_i \in \left\{ \{0\}, \mathbb{F}_2^n \right\}$.

In particular, in this case, bounds from activating S-boxes are optimal.

SPN Round: S-box layer



Possibility I

Algorithm

Algorithm

- Simply (de-)activate S-boxes
- Compute resulting subspace trail

Complexity (No. of starting Us)

For k S-boxes: 2^k (can be further decreased to k).

This approach is independent of the S-box, i. e. any S-box without linear structures behaves the same with respect to subspace trails.

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The problem with S-boxes that have linear structures

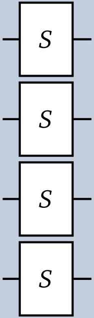
Subspace trails through S-box layers with *one*-linear structures are not necessarily a direct product of subspaces (see e. g. PRESENT).

Possibility II

S-boxes with linear structures

Observation

U

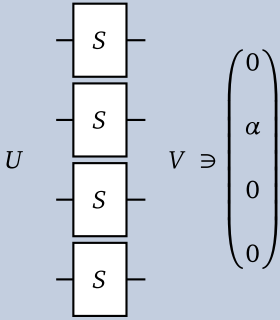


$$V \ni \begin{pmatrix} 0 \\ \alpha \\ 0 \\ 0 \end{pmatrix}$$

Possibility II

S-boxes with linear structures

Observation



Algorithm Idea

Compute the subspace trails for any starting point $W_{i,\alpha} \in \mathbb{W}$, with

$$W_{i,\alpha} := (\underbrace{0,\ldots,0}_{i-1},\alpha,0,\ldots,0)$$

Complexity (Size of $\mathbb{W}$)

For an S-box layer $F : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ with k S-boxes, each n-bit: $|\mathbb{W}| = k \cdot (2^n - 1)$

Conclusion/Questions

Thank you for your attention!

Main Result

- Provable bound length of *every possible* subspace trail in SPN cipher

Open Problems

- Other structures then SPNs?
- Truncated Differentials?

Mainboard & Questionmark Images: flickr

References I

[Knu94] L. R. Knudsen. “Truncated and Higher Order Differentials”. In: *FSE'94*. Vol. 1008. LNCS. Springer, 1994, pp. 196–211. doi: 10.1007/3-540-60590-8_16.

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[GRR16] L. Grassi, C. Rechberger, and S. Rønjom. “Subspace Trail Cryptanalysis and its Applications to AES”. In: *IACR Trans. Symmetric Cryptol.* 2016.2 (2016), pp. 192–225. doi: 10.13154/tosc.v2016.i2.192-225.