# Subspace Trail Cryptanalysis and its Applications to AES

**Lorenzo Grassi**, **Christian Rechberger and Sondre Rønjom**

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# Introduction

In the case of AES, several alternative representations (algebraic representation [**MR02**], dual ciphers of AES [**BB02**], super-box [**DR06**], twisted representation [**Gil14**], ...) have been proposed to highlight some aspects of its algebraic structure, differential nature, ...

We introduce Subspace Trail Cryptanalysis to formally and easily describe distinguishers and key-recovery attacks of AES-like cipher.

We believe that *the simplicity of the new representation can play a significant heuristic role in the investigation of structural attacks on AES-like cipher*.

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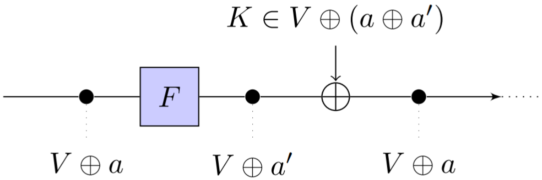
# Subspace Trail Cryptanalysis

# Invariant Subspace Cryptanalysis

If an invariant subspace $V$ exists such that

$$F_k(V \oplus a) = V \oplus a,$$

it is possible to mount distinguishers and key-recovery attacks (e.g. [**LAA+11**], [**LMR+15**], ...).




If no special symmetries or constants allow for invariant subspace, *can subspace properties still be used?*

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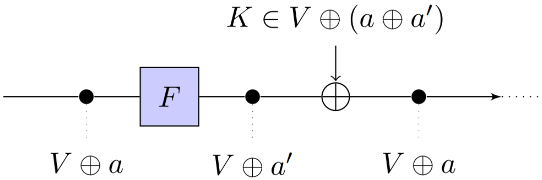


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# Subspace Trail

**Definition**

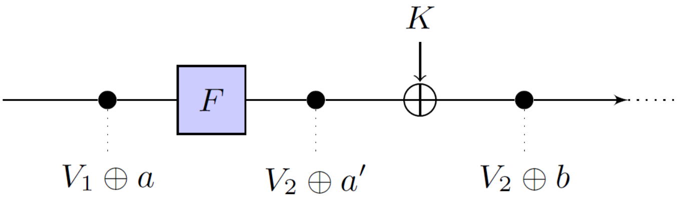
Let $(V_0, V_1, ..., V_r)$ denote a set of $r + 1$ subspaces with $\dim(V_i) \leq \dim(V_{i+1})$. If for each $i = 0, ..., r - 1$ and for each $a_i \in V_i^{\perp}$, there exists (unique) $a_{i+1} \in V_{i+1}^{\perp}$ such that

$$F(V_i \oplus a_i) \subseteq V_{i+1} \oplus a_{i+1},$$

then $(V_0, V_1, ..., V_r)$ is a subspace trail of length $r$ for the function F.

# Subspace Trail - Example

*Example of Subspace Trail* of length 1:




$\forall a \in V_1^\perp$ there exists $b \in V_2^\perp$ s.t.

$$F_k(V_1 \oplus a) \subseteq V_2 \oplus b.$$

# AES

High-level description of AES:

- block cipher based on a design principle known as *substitution-permutation network*;
- block size of 128 bits = 16 bytes, organized in a $4 \times 4$ matrix;
- key size of 128/192/256 bits;
- 10/12/14 rounds:

$$R^i(x) = k^i \oplus MC \circ SR \circ \text{S-Box}(x).$$

# Subspaces for AES

We define the following subspaces:

- *column space* $\mathcal{C}_I$;
- *diagonal space* $\mathcal{D}_I$;
- *inverse-diagonal space* $\mathcal{ID}_I$;
- *mixed space* $\mathcal{M}_I$.

# The Column Space

**Definition**

*Column spaces* $\mathcal{C}_i$ for $i \in \{0, 1, 2, 3\}$ are defined as

$$\mathcal{C}_i = \langle e_{0,i}, e_{1,i}, e_{2,i}, e_{3,i} \rangle.$$

E.g. $\mathcal{C}_0$ corresponds to the symbolic matrix

$$\mathcal{C}_0 = \left\{ \begin{bmatrix} x_1 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \\ x_4 & 0 & 0 & 0 \end{bmatrix} \middle| \forall x_1, x_2, x_3, x_4 \in \mathbb{F}_{2^8} \right\} \equiv \begin{bmatrix} x_1 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \\ x_4 & 0 & 0 & 0 \end{bmatrix}$$

# The Diagonal Space

**Definition**

*Diagonal spaces* $\mathcal{D}_i$ for $i \in \{0, 1, 2, 3\}$ are defined as

$$\mathcal{D}_i = SR^{-1}(\mathcal{C}_i) = \langle e_{0,i}, e_{1,(i+1)}, e_{2,(i+2)}, e_{3,(i+3)} \rangle.$$

E.g. $\mathcal{D}_0$ corresponds to symbolic matrix

$$\mathcal{D}_0 \equiv \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & 0 \\ 0 & 0 & 0 & x_4 \end{bmatrix}$$

for all $x_1, x_2, x_3, x_4 \in \mathbb{F}_{2^8}$.

# The Inverse-Diagonal Space

**Definition**

*Inverse-diagonal spaces* $\mathcal{ID}_i$ for $i \in \{0, 1, 2, 3\}$ are defined as

$$\mathcal{ID}_i = SR(\mathcal{C}_i) = \langle e_{0,i}, e_{1,(i-1)}, e_{2,(i-2)}, e_{3,(i-3)} \rangle.$$

E.g. $\mathcal{ID}_0$ corresponds to symbolic matrix

$$\mathcal{ID}_0 \equiv \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 \\ 0 & 0 & x_3 & 0 \\ 0 & x_4 & 0 & 0 \end{bmatrix}$$

for all $x_1, x_2, x_3, x_4 \in \mathbb{F}_{2^8}$.

# The Mixed Space

**Definition**

The *i-th mixed spaces* $\mathcal{M}_i$ for $i \in \{0, 1, 2, 3\}$ are defined as

$$\mathcal{M}_i = MC(\mathcal{ID}_i).$$

E.g. $\mathcal{M}_0$ corresponds to symbolic matrix

$$\mathcal{M}_0 \equiv \begin{bmatrix} 0x02 \cdot x_1 & x_4 & x_3 & 0x03 \cdot x_2 \\ x_1 & x_4 & 0x03 \cdot x_3 & 0x02 \cdot x_2 \\ x_1 & 0x03 \cdot x_4 & 0x02 \cdot x_3 & x_2 \\ 0x03 \cdot x_1 & 0x02 \cdot x_4 & x_3 & x_2 \end{bmatrix}$$

for all $x_1, x_2, x_3, x_4 \in \mathbb{F}_{2^8}$.

# Subspaces Trail for AES

## Definition

Let $I \subseteq \{0, 1, 2, 3\}$. The subspaces $\mathcal{C}_I$, $\mathcal{D}_I$, $\mathcal{ID}_I$ and $\mathcal{M}_I$ are defined as:

$$\mathcal{C}_I = \bigoplus_{i \in I} \mathcal{C}_i, \quad \mathcal{D}_I = \bigoplus_{i \in I} \mathcal{D}_i, \quad \mathcal{ID}_I = \bigoplus_{i \in I} \mathcal{ID}_i, \quad \mathcal{M}_I = \bigoplus_{i \in I} \mathcal{M}_i.$$

$\{\mathcal{D}_I, \mathcal{C}_I, \mathcal{M}_I\}$ is a subspace trail of AES of length 2.

# Subspace Trail for AES (1/2)

For each $a \in \mathcal{D}_I^\perp$, there exists unique $b \in \mathcal{C}_I^\perp$ s.t.

$$R(\mathcal{D}_I \oplus a) = \mathcal{C}_I \oplus b.$$

E.g.:

$$\mathcal{D}_0 \oplus a \xrightarrow{\text{S-Box}(\cdot)} \mathcal{D}_0 \oplus b \xrightarrow{\text{SR}(\cdot)} \mathcal{C}_0 \oplus c \xrightarrow{\text{MC}(\cdot)} \mathcal{C}_0 \oplus d \xrightarrow{\text{ARK}(\cdot)} \mathcal{C}_0 \oplus e$$

$$\begin{bmatrix} A & C & C & C \\ C & A & C & C \\ C & C & A & C \\ C & C & C & A \end{bmatrix} \xrightarrow{\text{S-Box}(\cdot)} \begin{bmatrix} A & C & C & C \\ C & A & C & C \\ C & C & A & C \\ C & C & C & A \end{bmatrix} \xrightarrow{SR(\cdot)} \begin{bmatrix} A & C & C & C \\ A & C & C & C \\ A & C & C & C \\ A & C & C & C \end{bmatrix} \xrightarrow{MC(\cdot)} \begin{bmatrix} A & C & C & C \\ A & C & C & C \\ A & C & C & C \\ A & C & C & C \end{bmatrix}$$

# Subspace Trail for AES (2/2)

For each $a \in \mathcal{C}_I^{\perp}$, there exists unique $b \in \mathcal{M}_I^{\perp}$ s.t.

$$R(\mathcal{C}_I \oplus a) = \mathcal{M}_I \oplus b.$$

E.g.:

$$\mathcal{C}_0 \oplus a \xrightarrow{\text{S-Box}(\cdot)} \mathcal{C}_0 \oplus b \xrightarrow{\text{SR}(\cdot)} \mathcal{ID}_0 \oplus c \xrightarrow{\text{MC}(\cdot)} \mathcal{M}_0 \oplus d \xrightarrow{\text{ARK}(\cdot)} \mathcal{M}_0 \oplus e$$

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# Part II

# Example of Use Case: Applications on AES

# Secret-Key Distinguisher up to 4 Rounds

*Re-describe - in a formal and easy way - Secret-Key Distinguisher up to 4 rounds* that exploit a property which is independent of the secret key:

- *Truncated Differential*
- *Impossible Differential*
- Integral

*using subspace trail notation*.

If $x, y \in \mathcal{X} \oplus a$, then $x \oplus y \in \mathcal{X}$.

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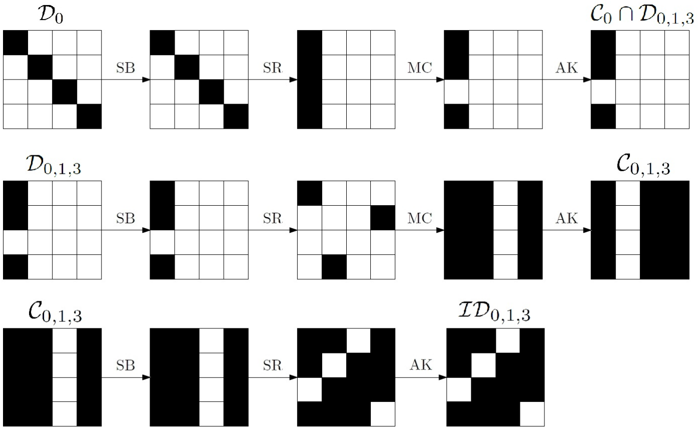
# Truncated Differential - 3-round AES




Equivalent to:

$$Prob[R^3(p^1) \oplus R^3(p^2) \in \mathcal{ID}_{0,1,3} \,|\, p^1 \oplus p^2 \in \mathcal{D}_0] = 2^{-32}.$$

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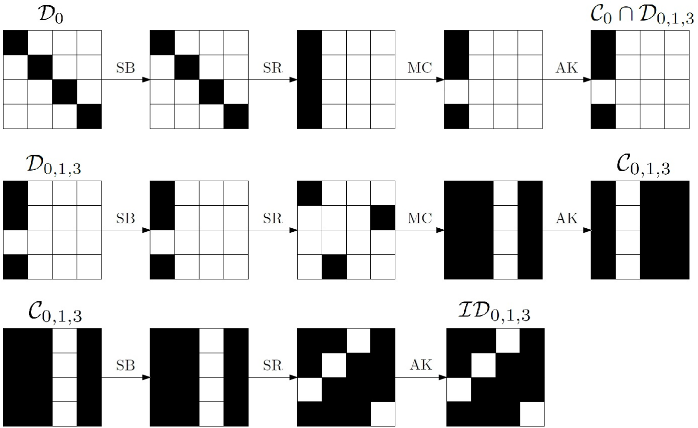


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# Truncated Differential on 3-round AES - *Comparison*

By A. Biryukov and D. Khovratovich [**BK07**]: *We will use a differential which starts with four active S-boxes at the 1st round. We choose those active S-boxes to appear in positions which arrive in one column after the ShiftRows transformation. Then with probability* $2^{-6}$ *four active S-boxes will collapse to three (one byte out of four getting a zero difference). After the second round the three active bytes are expanded into 12 active bytes and there will still remain 4 passive bytes. This differential can be schematically described as* $4 \to 3 \to 12$.

Let $I, J \subseteq \{0, 1, 2, 3\}$ with $|I| = 1$ and $|J| = 3$. For each $p^1, p^2$:

$$p^1 \oplus p^2 \in \mathcal{D}_I \xrightarrow[prob.\ 2^{-6}]{R(\cdot)} R(p^1) \oplus R(p^2) \in \mathcal{C}_I \cap \mathcal{D}_J \xrightarrow[prob.\ 1]{R^2(\cdot)} c^1 \oplus c^2 \in \mathcal{M}_J$$

where $c^1 = R^3(p^1)$ and $c^2 = R^3(p^2)$.

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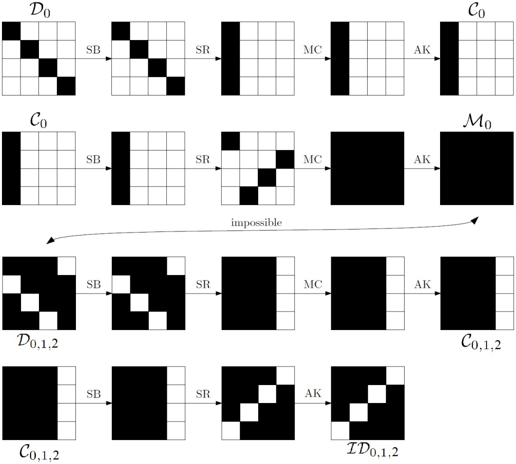
# Truncated Differential on 3-round AES - *Statement*

*Given a pair of plaintexts which differ by* $1 \leq d \leq 3$ *diagonals (the plaintexts are equal in the other diagonals), what is the probability that after 3-round the corresponding ciphertexts are equal in* $1 \leq n \leq 3$ *anti-diagonals?*

For each $I, J \subseteq \{0, 1, 2, 3\}$ and for each $p^1, p^2$:

$$Prob[R^3(p^1) \oplus R^3(p^2) \in \mathcal{M}_J \,|\, p^1 \oplus p^2 \in \mathcal{D}_I] = (2^8)^{-4|I| + |I| \cdot |J|}.$$

# Impossible Differential - 4-round AES




Equivalent to:

$$Prob[R^4(p^1) \oplus R^4(p^2) \in \mathcal{ID}_{0,1,2} \,|\, p^1 \oplus p^2 \in \mathcal{D}_0] = 0.$$

# Impossible Differential on 4-round AES - *Comparison*

By E. Biham and N. Keller [**BK00**]: *If a pair of plaintexts differ by only one byte then the ciphertexts cannot be equal in any of the following combinations of bytes: (1,6,11,16), (2,7,12,13), (3,8,9,14), nor (4,5,10,15).*

Let $p^1 \neq p^2$. For each $I, J, H \subseteq \{0,1,2,3\}$ with $|I| = |H| = 1$ and $|J| = 3$:

$$Prob[R^4(p^1) \oplus R^4(p^2) \in \mathcal{M}_J \,|\, p^1 \oplus p^2 \in \mathcal{D}_I \cap \mathcal{C}_H] = 0.$$

*More generally*, for each $I, J \subseteq \{0,1,2,3\}$ with $|I| + |J| \leq 4$:

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The reasons are:

- $\mathcal{D}_J \cap \mathcal{M}_I = \{0\}$ for all $I, J$ with $|I| + |J| \leq 4$, i.e. $Prob[x \in \mathcal{D}_J \,|\, x \in \mathcal{M}_I] = 0$;
- for all $a$ and for all $J$, there exists $b$ s.t. $R^2(\mathcal{D}_J \oplus a) = \mathcal{M}_J \oplus b$, that is $Prob[R^2(p^1) \oplus R^2(p^2) \in \mathcal{M}_J \,|\, p^1 \oplus p^2 \in \mathcal{D}_J] = 1$.

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# First Applications

- New key-dependent 5-round distinguisher: Complexity $2^{96}$ (best before: $2^{128}$ at Crypto 2016 by Sun, Liu, Gou, Qu and Rijmen [**SMG+16**]).
- Key-recovery with known S-Box: Truncated Differential-style attacks similar in complexity with the current best MitM-style attacks [**BDD+12**]-[**BDF11**] for up to 4 rounds.
- Key-recovery with secret S-Box: not competitive but with a new twist.

# Part III

## Key-Recovery Attacks on AES with a single Secret S-Box

# AES with a single Secret S-Box

Consider AES with a single secret S-Box: the size of the secret information increases from 128-256 bits to 1812-1940.

*How does the security of the AES change when the S-Box is replaced by a secret S-Box, about which the adversary has no knowledge?*

# AES with a single Secret S-Box

For all the attacks ([**BS01**], [**TKK+15**], ...) in literature:

1. determine the secret S-Box up to additive constants, i.e. S-Box$(a \oplus x) \oplus b$;
2. exploit this knowledge to find the key.

*Is it possible to find directly the key, i.e. without finding or exploiting any information of S-Box?*

*Yes*: exploit the fact that each row of the MixColumns matrix has two identical elements.

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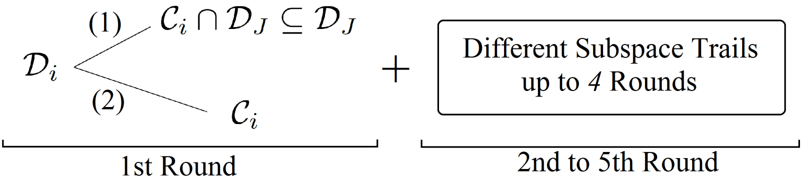
*Yes*: exploit the fact that each row of the MixColumns matrix has two identical elements.

# Attacks on AES with a single Secret S-Box - Details

Guess part of the key $\delta$, and consider a set of plaintexts $V_\delta \subset \mathcal{D}_i \oplus a$ which depends on $\delta$:

1. If $\delta$ is correct, then $R(V_\delta) \subseteq \mathcal{C}_i \cap \mathcal{D}_J \oplus b \subseteq \mathcal{D}_J \oplus b$ with prob. 1;
2. If $\delta$ is wrong, then $R(V_\delta) \subseteq \mathcal{C}_i \oplus c$ with prob. 1 and $R(V_\delta) \subseteq \mathcal{D}_J \oplus d$ with prob. *strictly less* than 1.



# Part IV

# Summary

# Summary and Open Problems

- *Subspace Trail Cryptanalysis*: a formal notation that includes techniques based on impossible or truncated differentials and integrals as special cases;
- Various New Key-Recovery Attacks on reduced AES;
- Open Problem: more applications where mixed view of e.g. differential and integral properties makes sense.

# Follow-Up Work

Stay tuned for

"*A New Structural-Differential Property of 5-Round AES*"

at Rump Session (to appear at Eurocrypt 2017 [**GRR17**]).

*"Consider AES reduced to 5 rounds. Given $2^{32\cdot|I|}$ plaintexts in the same coset of a diagonal space $\mathcal{D}_I$ for $I \subseteq \{0,1,2,3\}$, the number of different pairs of ciphertexts that belong to the same coset of a mixed space $\mathcal{M}_J$ for $J \subseteq \{0,1,2,3\}$ is a multiple of 8 with probability 1, independently of the secret-key, of the details of the S-Box and of the MixColumns matrix (with the exception that its branch number is 5)."*

Thanks for your attention!

Questions?

Comments?

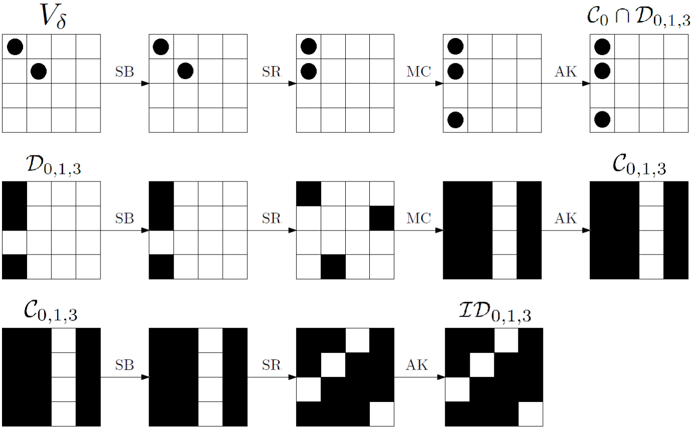
# Key-Recovery Attack on 3-round AES

$$V_\delta = \{(p^i, c^i) \quad \forall i = 0, ..., 2^8 - 1 \mid p^i_{0,0} \oplus p^i_{1,1} = \delta \\ \text{and} \quad p^i_{k,l} = p^j_{k,l} \quad \forall (k,l) \neq \{(0,0),(1,1)\} \text{ and } \forall i \neq j\}.$$

Since $MC_{0,0} = MC_{1,1}$, attack on 3 rounds:

- If $\delta$ is correct, given $p^1, p^2 \in V_\delta$ then $R^3(p^1) \oplus R^3(p^2) \in \mathcal{M}_J$ with prob. 1;
- If $\delta$ is wrong, given $p^1, p^2 \in V_\delta$ then $R^3(p^1) \oplus R^3(p^2) \in \mathcal{M}_J$ with prob. $2^{-8}$.

# Example: Attack on 3-round AES with secret S-Box

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Since $MC_{0,0} = MC_{1,1}$, attack on 5 rounds:

- If $\delta$ is correct, given $p^1, p^2 \in V_\delta$ then $R^5(p^1) \oplus R^5(p^2) \in \mathcal{M}_J$ with prob. 0;
- If $\delta$ is wrong, given $p^1, p^2 \in V_\delta$ then $R^5(p^1) \oplus R^5(p^2) \in \mathcal{M}_J$ with prob. $2^{-94}$.

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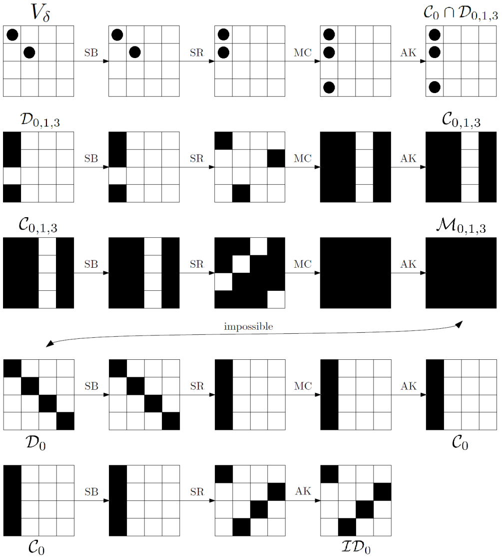

# Attacks on AES with secret S-Box - Results

| Attack | Rounds | Data | Cost | Memory |
|---|---|---|---|---|
| **Trunc. Diff.** | **2.5 - 3** | **$2^{13.6}$ CP** | **$2^{13.2}$ XOR** | **small** |
| SASAS [**BS01**] | 2.5 | $2^{16}$ CP | $2^{21}$ E | $2^{16}$ |
| **Integral** | **2.5 - 3** | **$2^{19.6}$ CP** | **$2^{19.6}$ XOR** | **small** |
| Integral$^\star$ [**TKK+15**] | 3.5 - 4 | $2^{16}$ CC | $2^{17.7}$ E | $2^{16}$ |
| Integral$^\star$ [**TKK+15**] | 3.5 - 4 | $2^{16}$ CP | $2^{28.7}$ E | $2^{16}$ |
| **Trunc. Diff** | **3.5 - 4** | **$2^{30}$ CP** | **$2^{29.7}$ E** | **$2^{30}$** |
| Integral$^\star$ [**TKK+15**] | 4.5 - 5 | $2^{40}$ CC | $2^{38.7}$ E | $2^{40}$ |
| Integral$^\star$ [**TKK+15**] | 4.5 - 5 | $2^{40}$ CP | $2^{54.7}$ E | $2^{40}$ |
| **Imp. Diff.** | **4.5 - 5** | **$2^{102}$ CP** | **$2^{100.4}$ E** | **$2^{8}$** |
| Integral [**SMG+16**] | 5 | $2^{128}$ CC | $2^{129.6}$ XOR | small |

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