On Boomerang Attacks on Quadratic Feistel Ciphers

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Boomerang Distinguishers

Differential Distinguisher

(α, β) so that $\quad E(M_1 + \alpha) + E(M_1) = \beta \quad$ with high probability

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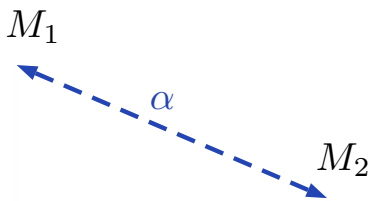

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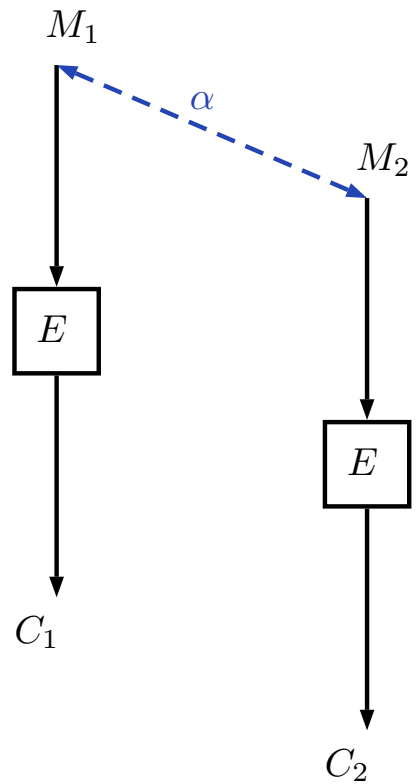

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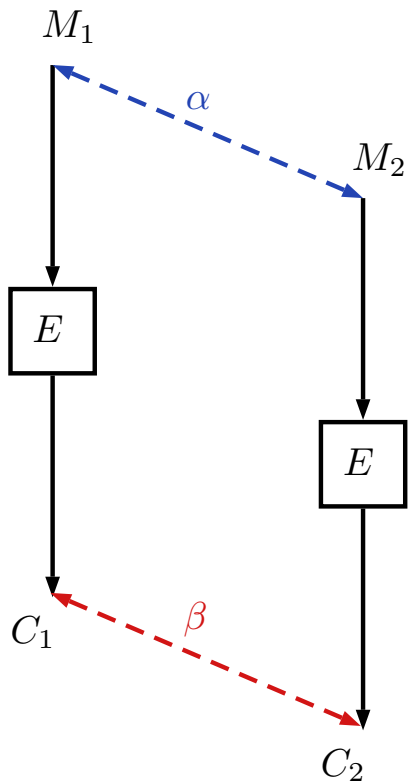

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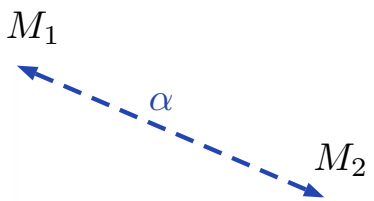

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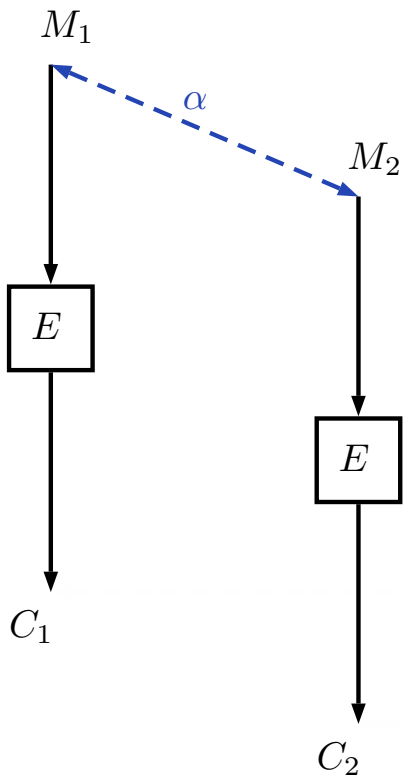

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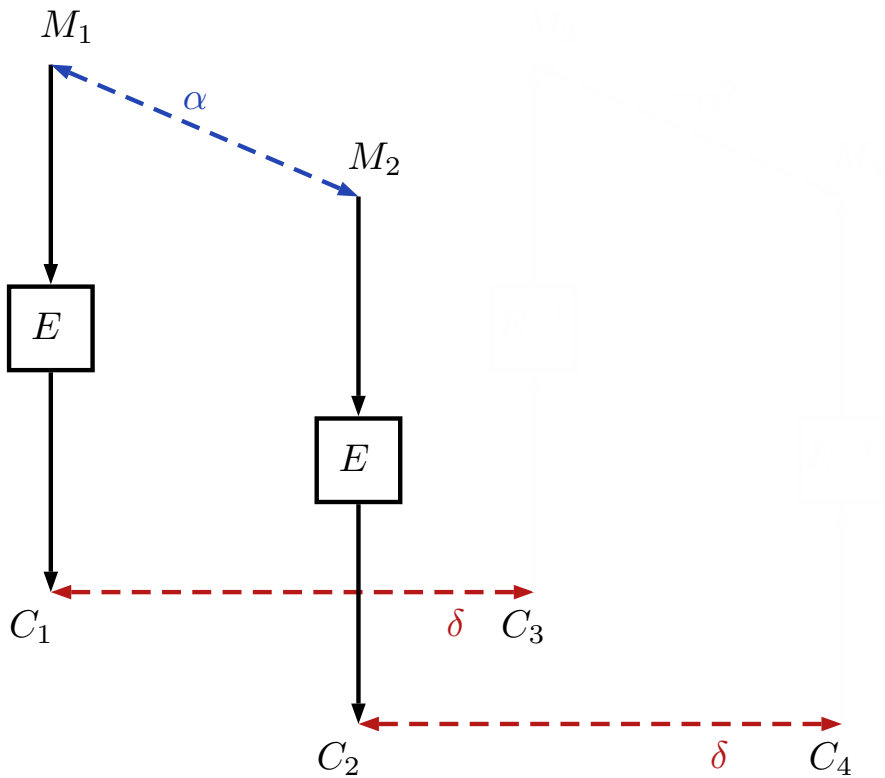

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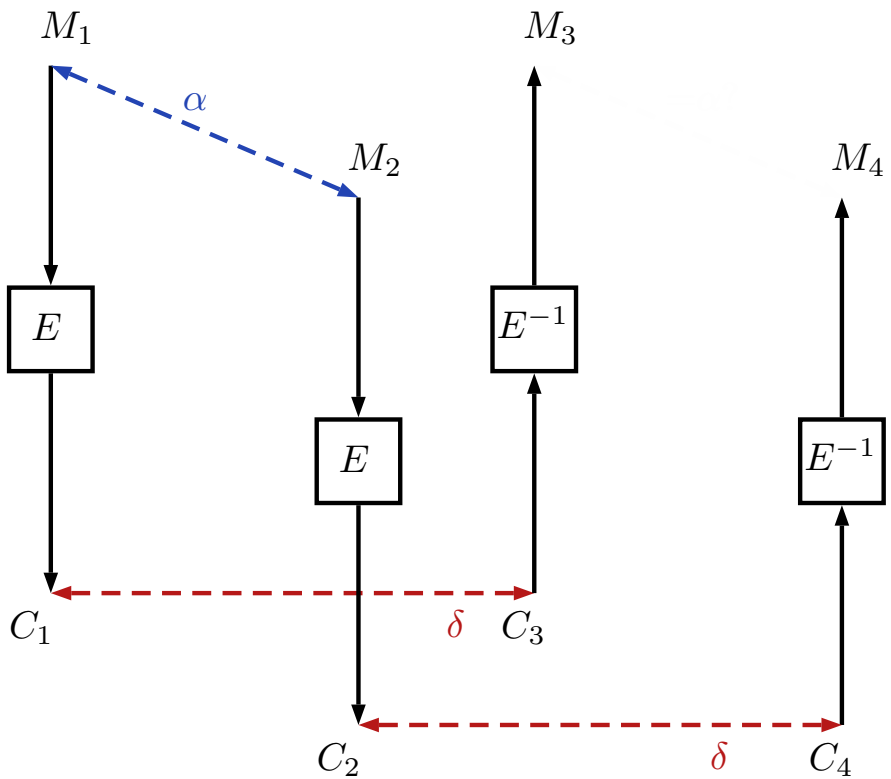

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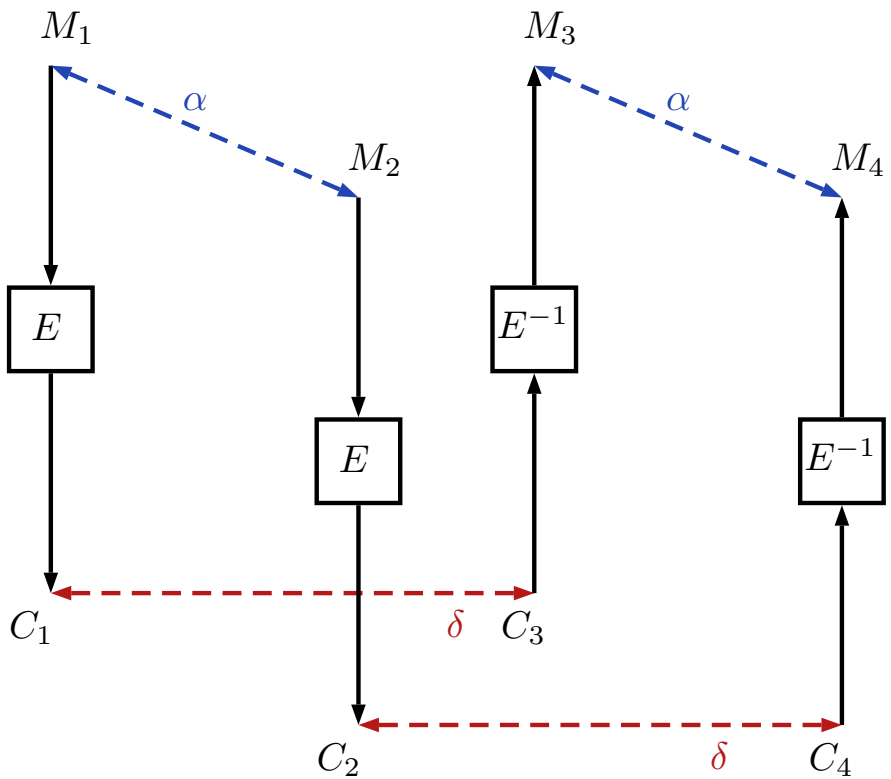

Building a Boomerang Distinguisher

- Rewrite $E = E_1 \circ E_0$
- Find good differentials:
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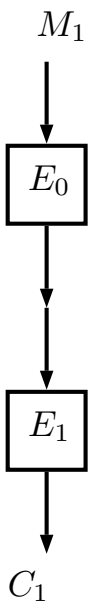
Probability:



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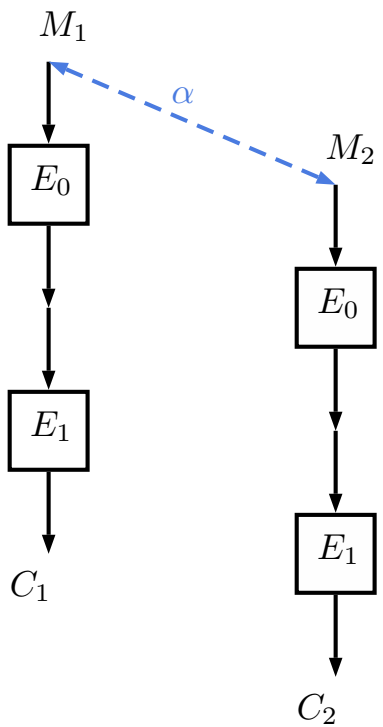

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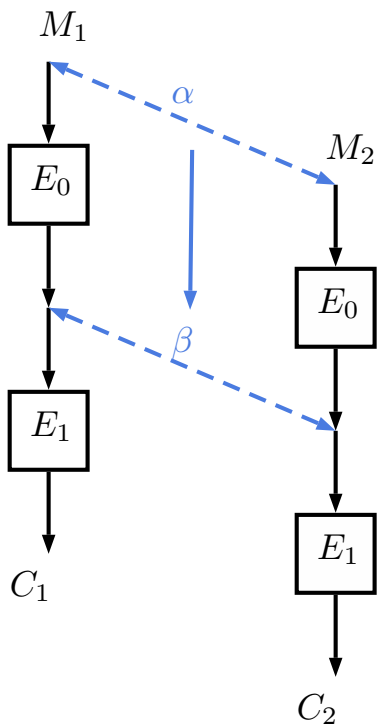

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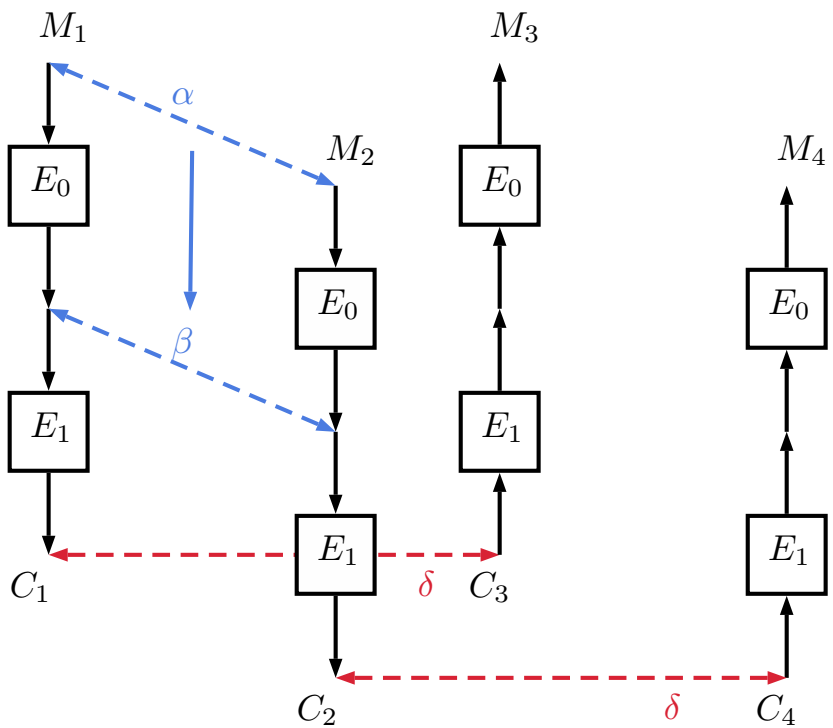

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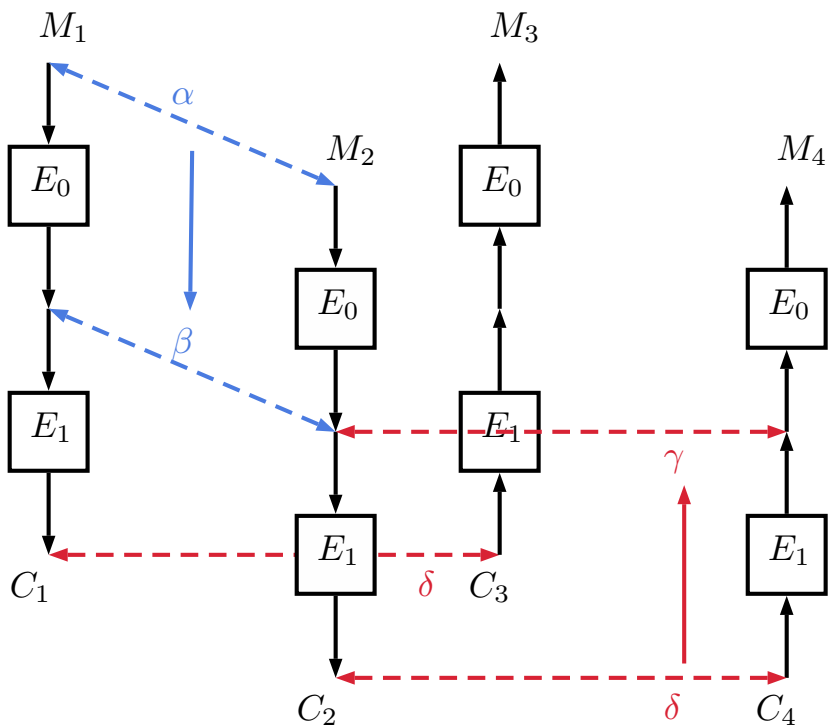
Probability:

$p \times q$



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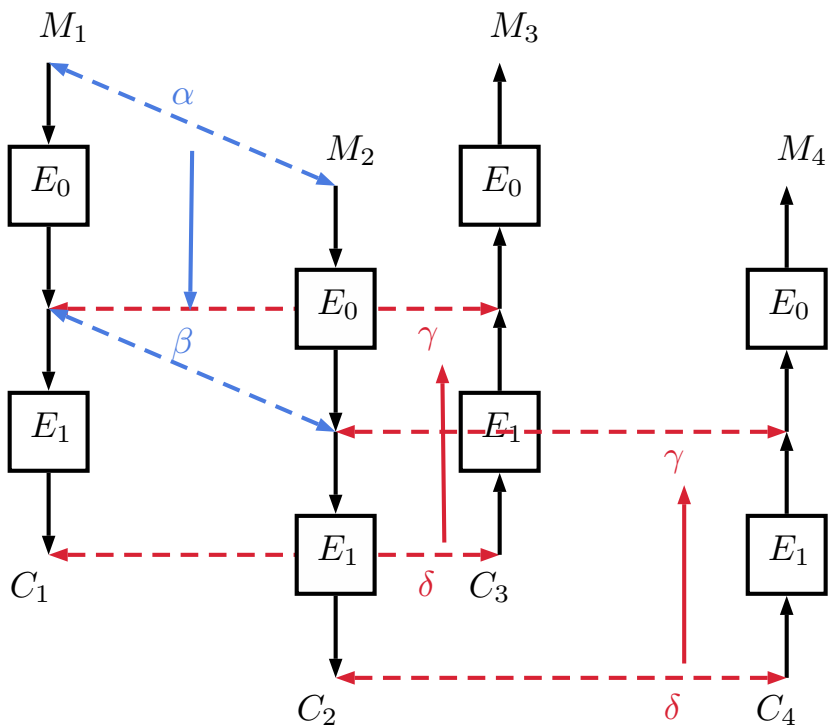

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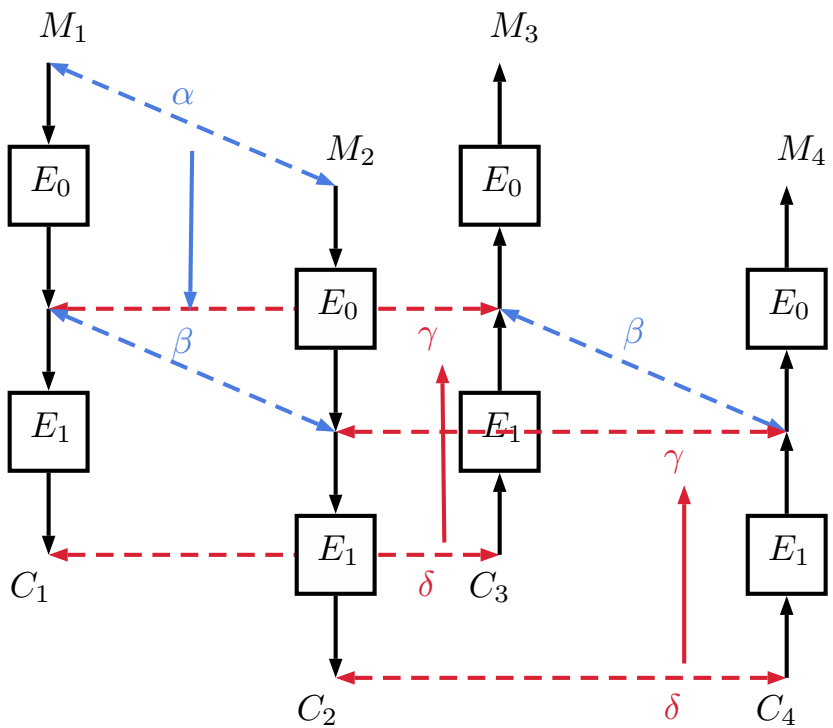

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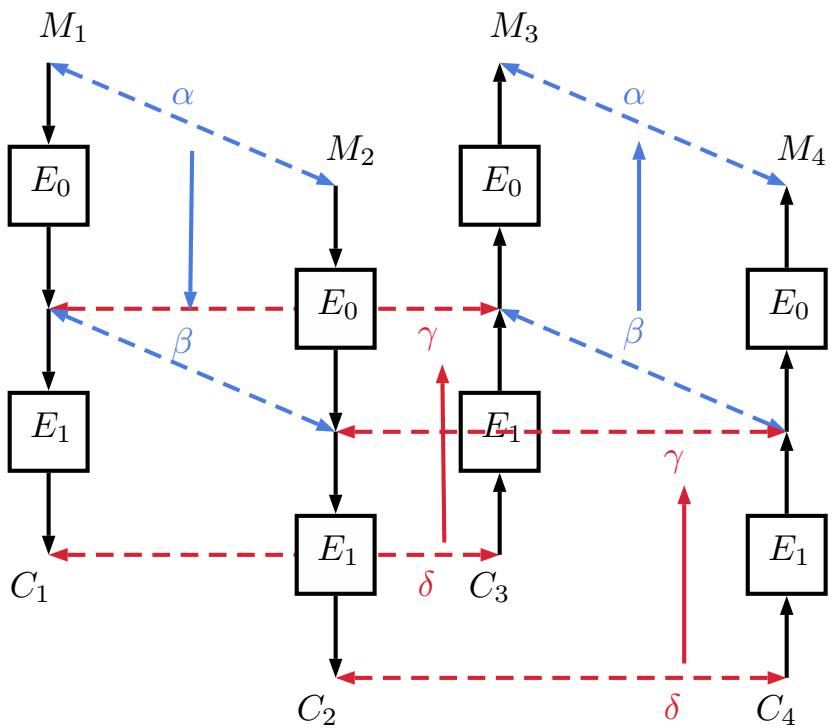

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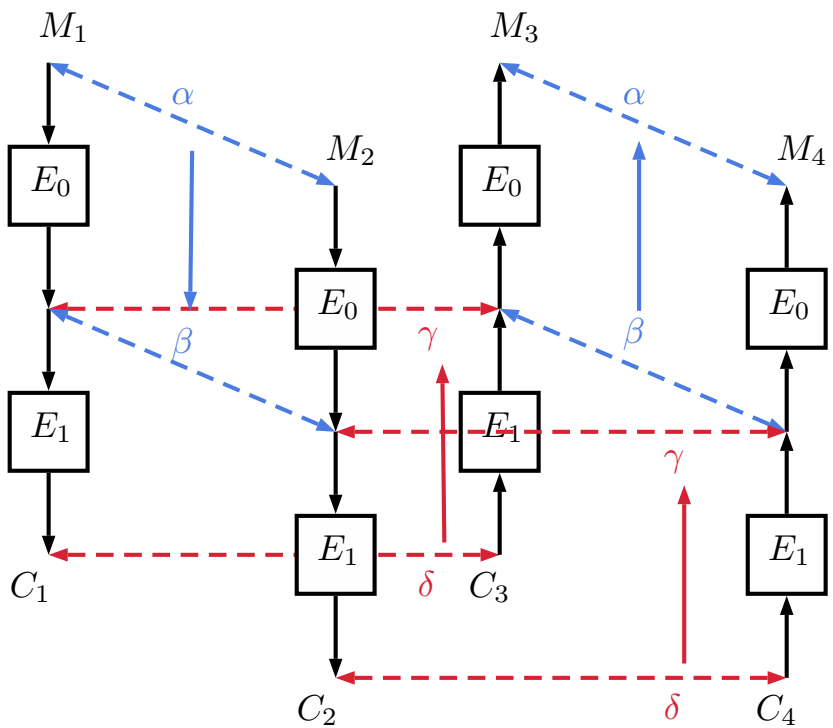
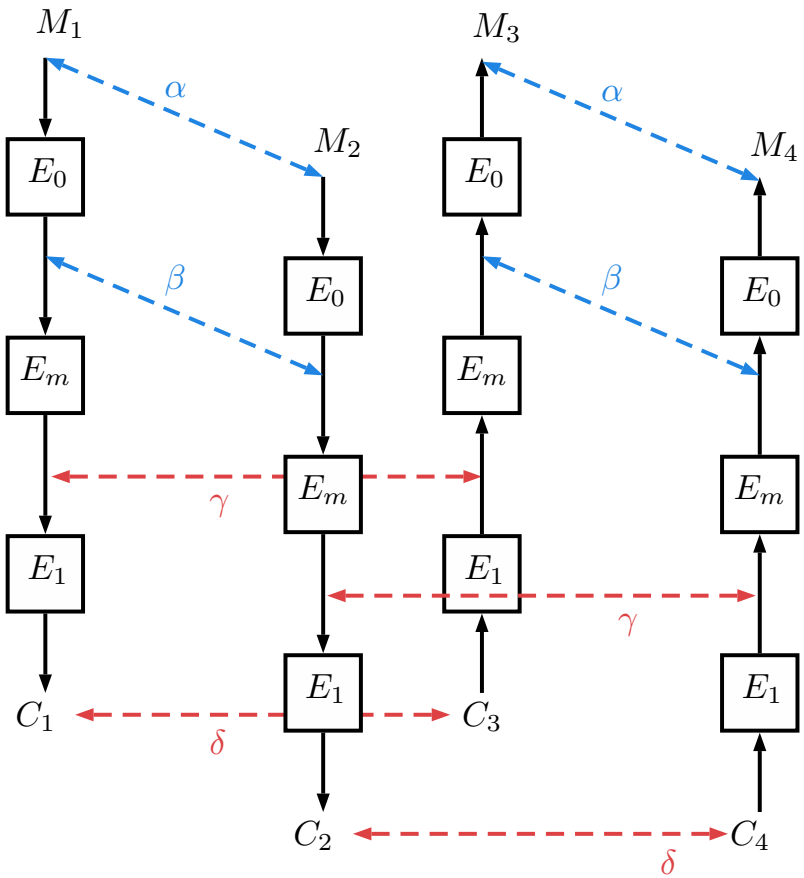


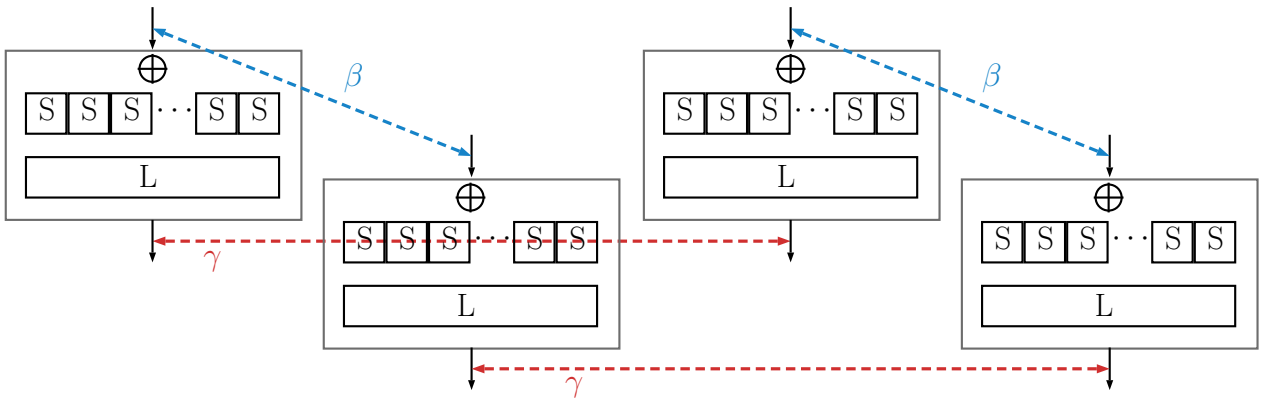
Expected probability of p^2q^2 if all the events are **independent**

Careful Handling of the Middle: the Sandwich Framework



- $E = E_1 \circ E_m \circ E_0$
- estimated probability $p^2 \times r \times q^2$
- $r = Pr[E_m^{-1}(E_m(x_1) \oplus \gamma) \oplus E_m^{-1}(E_m(x_1 \oplus \beta) \oplus \gamma) = \beta]$

BCT approach



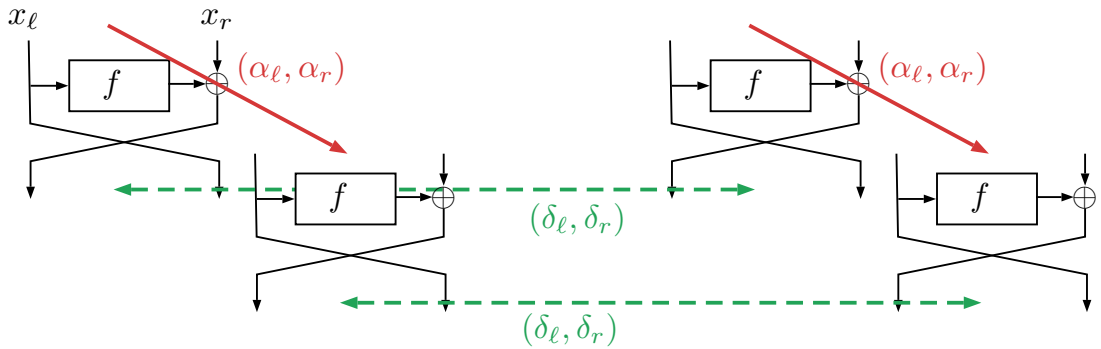
Systematic approach to compute boomerang probabilities for 1 round of SPN. Reduces to analyzing the Sbox.

Many variations:

- More rounds
- Different kind of cipher

Quadratic Feistel Boomerangs

Generic 1-round Boomerang on a Feistel cipher



A boomerang returns from 1 round of Feistel cipher with round function f for the input x_ℓ, x_r if and only if

$$f(x_\ell) \oplus f(x_\ell \oplus \delta_r) \oplus f(x_\ell \oplus \alpha_\ell) \oplus f(x_\ell \oplus \delta_r \oplus \alpha_\ell) = 0,$$

that is, the **second derivative** of f at points α_ℓ, δ_r must be zero.

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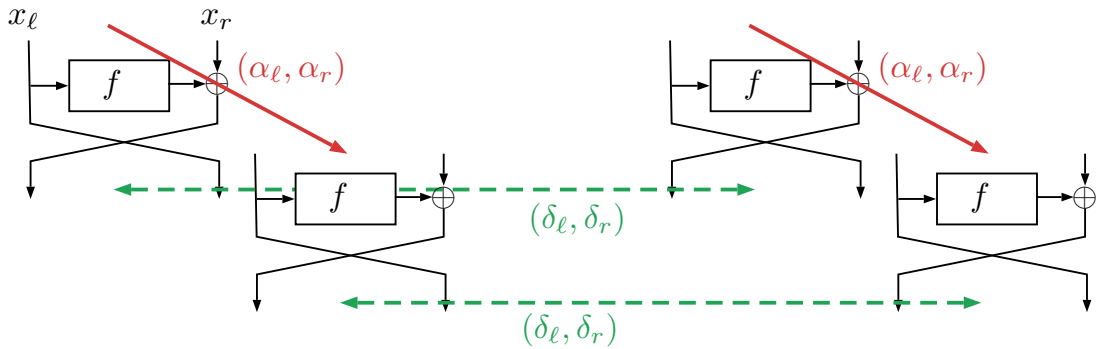

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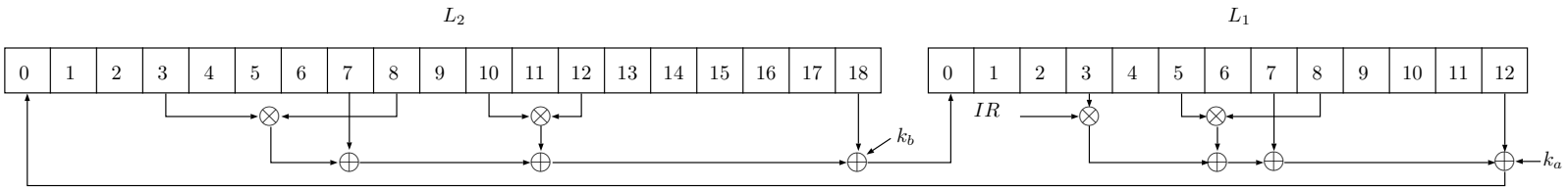
If f is a quadratic function, its second order derivative is a constant.

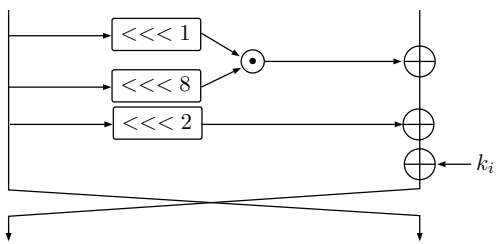
Either the boomerang always returns, or never

Concrete study

KATAN



Simon



- Many variants, from 32 bits size to 64 (KATAN) or 128 (Simon)
- Both Feistel-like
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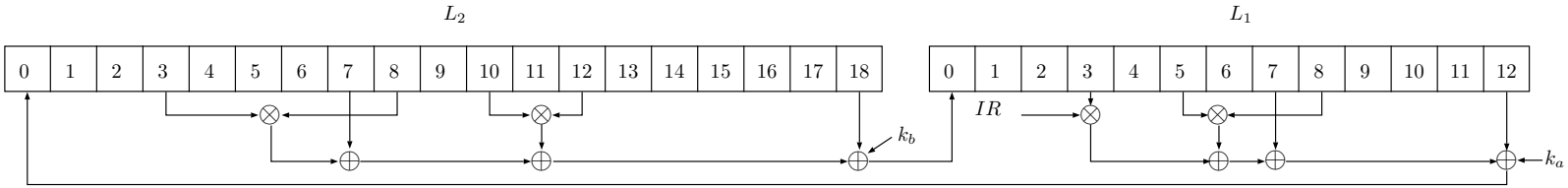

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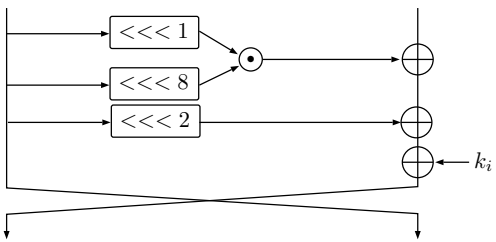

Boomerang constraint for KATAN32

$$\begin{cases} \alpha_{12} \cdot \delta_{11} \oplus \alpha_{10} \cdot \delta_{13} \oplus \alpha_8 \cdot \delta_4 \oplus \alpha_3 \cdot \delta_9 = 0 \\ \alpha'_8 \cdot \delta'_6 \oplus \alpha'_5 \cdot \delta'_9 = 0 \end{cases}$$

Boomerang constraint for Simon

$$(\alpha_\ell \lll 8) \cdot (\delta_r \lll 1) \oplus (\alpha_\ell \lll 1) \cdot (\delta_r \lll 8) = 0$$

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- Identified 20 distinguishers in 6 articles
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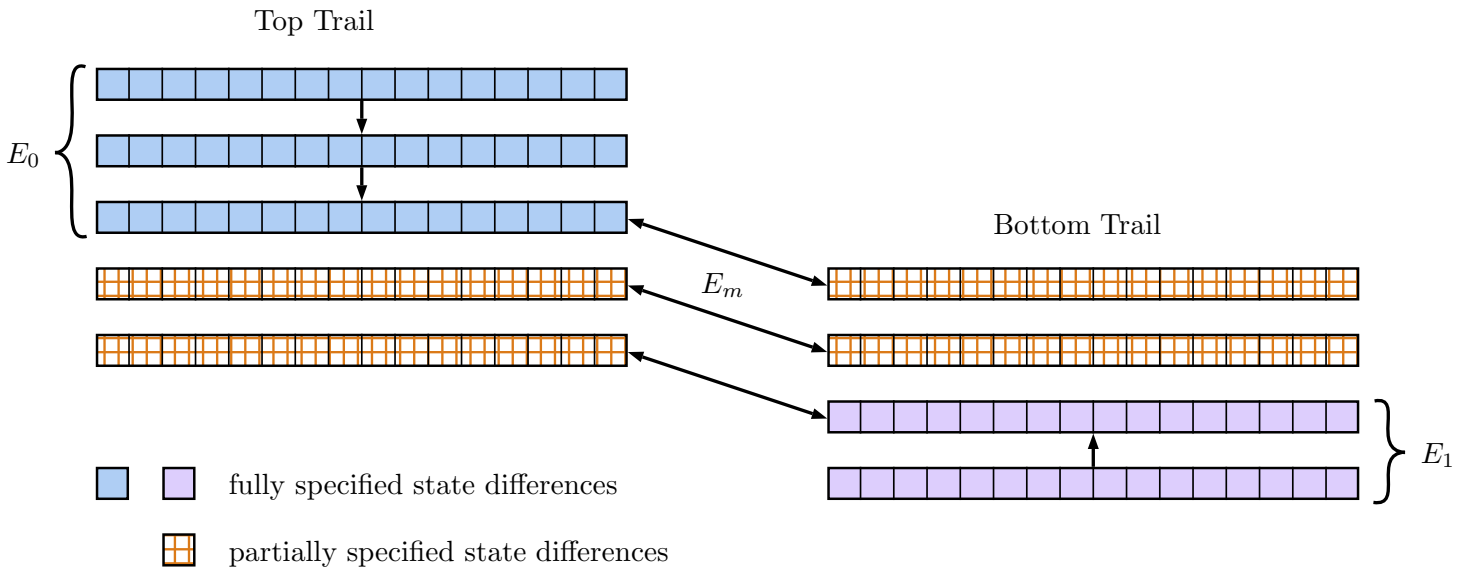
- Probabilities are inaccurate
- In most cases, the distinguisher doesn't work

New Distinguishers and Attacks

Our SMT Model



- E_0 and E_1 are considered independent one from the other
  - **fully specified** differential trails
  - p^2 and q^2 probability
- middle rounds cover the interactions
  - boomerang constraints are enforced
  - Truncated differentials $(0, 1, ?)$
  - the model can fix some non-deterministic transitions

Results of our SMT model for RK Boomerang Distinguishers on KATAN 32

Rounds	140	141	142	143	144	145	146	147	148	149	150	151	152
Top	40	40	41	41	42	40	43	43	44	45	45	45	46
Middle	60	61	60	61	60	65	60	61	60	59	60	61	60
Bottom	40	40	41	41	42	40	43	43	44	45	45	45	46
Model proba.$^\diamond$	-17	-17	-20	-21	-22	-23	-25	-25	-27	-29	-31	-31	-33
Exp. proba.$^\diamond$	-16.2	-15.8	-20.2	-19.8	-19.1	-22.5	-24.3	-24.3	-26.3	-28.3	-30.2	-30.1	-31.7

$\diamond$ Binary logarithm of the probabilities.

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Best attack [RR16]

206 rounds, MITM, Single-key

Rotational-Xor and Rotational-Xor Differential Rectangles

Normal Differences

$$x \oplus x' = \alpha$$

Rotational-Xor differences

$$x \oplus (x' \lll 1) = \alpha$$

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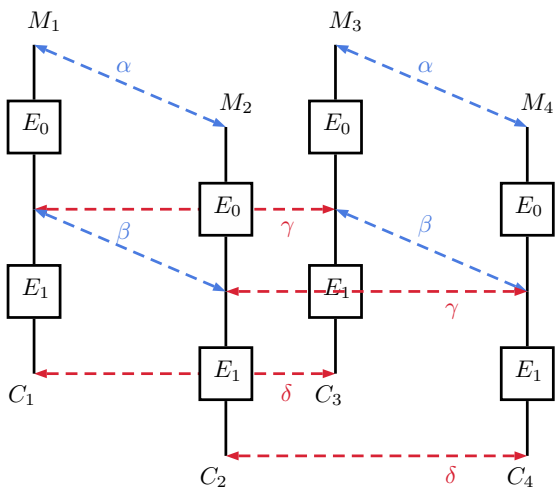
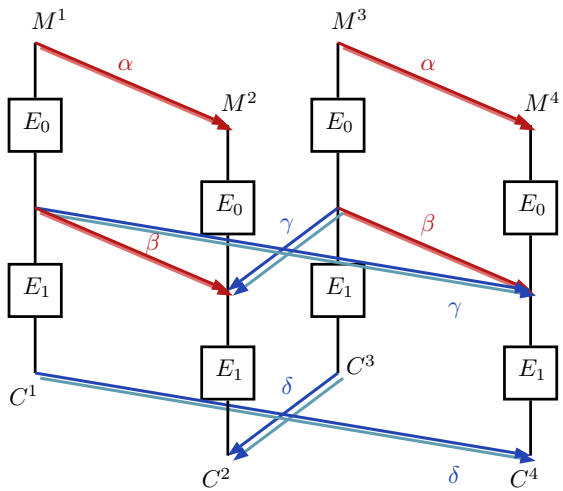
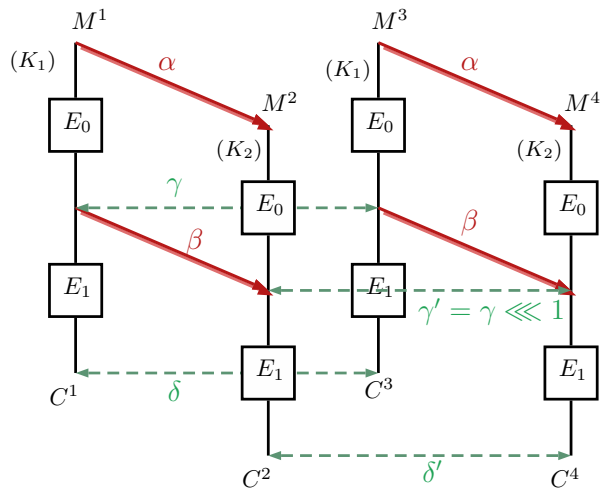


Normal Boomerang



Rotational-Xor Boomerang



Rotational-Xor Differential Boomerang

Results of our SMT model on Simon-32/64

Related-key boomerang distinguishers

Rounds	12	13	14	15	16	17
Cut	5+2+5	5+3+5	5+4+5	6+3+6	5+6+5	6+5+6
Model proba.	0	-3	-7	-11	-19	-25
Experimental proba.	0	-2.7	-6.7	-10.4	-18.8	-23.6

RX-boomerang distinguishers

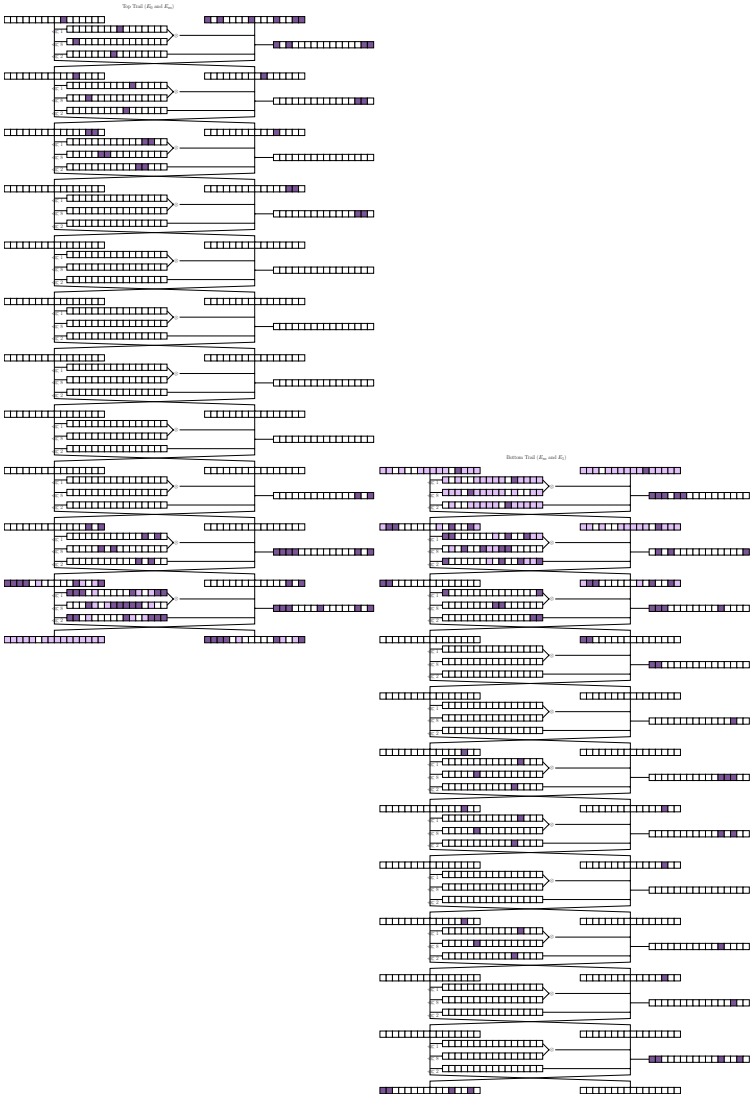
Rounds	13	14	15	16	17	18	19
Starting round	3	3	10	3	3	3	3
Cut	4+5+4	5+4+5	5+5+5	5+6+5	6+5+6	6+6+6	7+5+7
Model proba.	0	-3	-6	-12	-16	-24	-30
Experimental proba.	0	-3	-6	-12	-16	-24	-29.5

RX-differential boomerang distinguishers

Rounds	13	14	15
Starting round	3	3	3
Cut	7+4+2	5+5+4	6+5+4
Model proba.	-17	-23	-28
Experimental proba.	-17.2	-21	-27.6

(Binary logarithm of the probabilities)

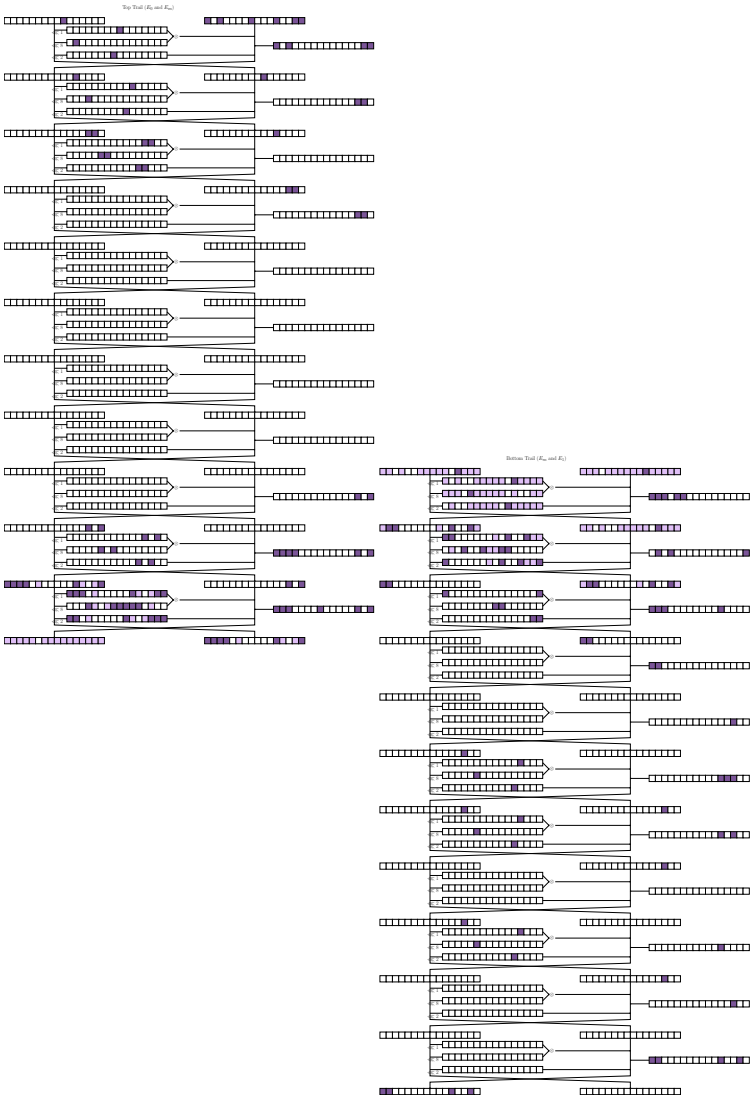
Attacks against Simon32



- Start from an RX-boomerang distinguisher
- Add 3 rounds above and below
- Results:

Rounds	Data	Time
24	2^{31}	$2^{54.6}$
25	2^{34}	$2^{59.7}$

Attacks against Simon32



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- Add 3 rounds above and below
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Rounds	Data	Time
24	2^{31}	$2^{54.6}$
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Previous best attack [CCW+18]

24 Rounds, Linear, Single key, 2^{32} Data

Conclusion

Summary

- Found issues in most boomerangs against KATAN and Simon
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Advices to cryptanalysts

- Don't be naive with boomerangs!
- If possible, verify experimentally